Duality and the cosmological constant[1]

Hadi Salehi[2]

Institute for Studies in Theoretical Physics and Mathematics,
P.O.Box 19395-1795, Niavaran-Tehran, Iran
and
Arnold Sommerfeld Institute for Mathematical Physics, TU Clausthal,
Leibnizstr. 10, D-38678 Clausthal-Zellerfeld, Federal Republic of Germany

Abstract

A dynamical theory is studied in which a scalar field ϕ in Einstein-Minkowski space is coupled to the four-velocity N_μ of a preferred inertial observer in that space. As a consistent requirement on this coupling we study a principle of duality invariance of the dynamical mass-term of ϕ at some universal length in the small-distance regime. In the large-distance regime duality breaking can be introduced by giving a back-ground value to ϕ and a back-ground direction to N_μ. It is shown that, in an appropriate approximation, duality breaking can be related to the emergence of a characteristic phase in which the condensation of the ground state allows massive excitations with a characteristic scale of squared mass which agrees with present observational bound for the cosmological constant.

I-Introduction

In quantum field theory the structure of the vacuum is recognised to be interrelated with the condensation of scalar fields, a phenomenon characterised by remnant constant vacuum expectation values for those fields. In general, such scalar (vacuum) condensates

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[2] e-mail address: salehi@netware2.ipm.ac.ir

carry informations about various mass scales which are characteristic of the dynamical properties of the vacuum. For example, a particular type of massive particle excitation of vacuum can be characterised by the mass scale corresponding to the massive excitations around a given scalar condensate.
The condensation of the vacuum and the related appearance of characteristic mass scales may have an important effect on the energy content of the vacuum, because one expects that the cosmological constant receives potential contributions from any scale of mass which can be extracted from the mass spectrum of physical fields in quantum field theory. In particular, any massive particle excitation around a given scalar condensate can provide a particular type of contribution to the total value of the cosmological constant via the corresponding mass scale. In this way a non-vanishing contribution to the total energy density of vacuum may arise from the mass scale of any scalar condensate.
This kind of contribution, however, meets with an immediate difficulty. For example, the isolated contribution of the mass scale of the Higgs field in the standard model predicts a value for the cosmological constant which is many orders of magnitude away from the observational bound of that constant.
A solution of this problem which is a particular manifestation of the the cosmological constant problem, see [1] and references therein, may really require a theoretical scheme in which explicit recognization is given to the expected sensitivity of the total value of the cosmological constant to the entire mass spectrum of physical fields in quantum field theory, because in a unified theory the latter fields may not be independent so that unexpected cancellations among various contributions of the corresponding mass scales may occur. Such a theoretical scheme is likely to reduce the issue of the cosmological constant to a picture in which the consistent contribution to the total value of that constant comes from a preferred mass scale of the vacuum, namely that assigning a cosmological range to the massive excitations of the vacuum.
It must be emphasized that this picture is very subtile, and it is extremely difficult to find out how the contribution of the preferred mass scale among the other potential contributions could be established. But a preliminary task is to focus on the the nature of this preferred mass scale. In the present note we discuss tentative first steps in this direction.

II-The broken phase of Lorentz invariance

We remark that, in an exact Lorentz invariant vacuum, the energy density is, almost by definition, zero. Therefore, one should expect that the defining characteristic of a non-vanishing energy density in vacuum is a principal violation of Lorentz invariance. The nature of the preferred mass scale in question depends on the consistency of such a picture in an essential way.
To be more specific we remark that a principal violation of Lorentz invariance may act in

the right way to yield a consistent contribution to the cosmological constant via the mass scale of massive excitations around an associated scalar condensate. Before we present a model along this line, it is necessary to collect some general facts concerning what is expected to be the right way to think of a principal violation of Lorentz invariance.
Such a violation of Lorentz invariance should in fact be a consequence of the still unknown principles underlying the unification of quantum physics and gravity and is expected to manifest itself at some characteristic scale in the ultrashort distance regime, described by an absolute scale of length l_0. The understanding of the relation of l_0 to the Planck length is an elusive task for quantum gravity. Here we merely note that the length l_0 is expected to act as a sort of universal length that determines a lower bound to any scale of length probed in a measurement process.
It should be noticed that the existence of such a universal length in the small-distance regime is in contrast with the universal requirement of Lorentz invariance. Indeed, no absolute line of demarcation between small distances and large ones can be defined without having a positive definite measure of distances, a feature which is apparently absent in Einstein-Minkowski space.
It was pointed out by Blokhintsev [2] that, associating to the Einstein-Minkowski space a time-like vector N_μ, the so called internal vector, it is possible to distinguish between small distances and large ones by taking the positive definite interval

$$R^2 = (2N_\mu N_\nu - \eta_{\mu\nu})x^\mu x^\nu, \quad N_\mu N^\mu = 1 \tag{1}$$

Given such a metric, we may determine the absolute size of a distance by comparing R with the universal length l_0.
Generally, one would expect the idea of a universal length and the related internal vector to play a vital role in the ultrashort distance extrapolation of physics. Unfortunately, the practical need of this idea in the development of our present day physical concepts (to a large extend) disappeared with the achievement of renormalizable theories, which explains why that idea has not attracted wide attention in the particle physics community.
Different meanings are assigned in the literature to an internal vector, for a review see [3]. We shall adopt the most obvious interpretation of the vector N_μ and consider N_μ at each space-time point as characterising the four velocity of a preferred inertial observer. The assignment of N_μ to the vacuum singles out then a coordinate system as preferred, namely that in which the preferred inertial observer is at rest. Correspondingly, only a special group of Lorentz transformations can have an intrinsic significance, namely those which leaves the special form $N_\mu = (1, 0, 0, 0)$ invariant. Physically, this special group can be thought of as being composed of those transformations which move a physical system without affecting the preferred rest frame. Thus, if one limits oneself to this special group, the internal vector appears as a universal field which has the same absolute value in vacuum. In this respect the internal vector can be considered as corresponding to a characteristic property of the vacuum in the the broken phase of Lorentz invariance.
We shall not comment here on the probable form of the preferred rest frame defined by N_μ, because the results we want to present depend only on general considerations[3].

[3] For the conceivable form of such a rest frame in a laboratory on the earth a proposal is made in [4] where possible experiments to detect it are also discussed.

III-The Model

We first study a theory which relates the broken phase of Lorentz invariance characterised by an internal vector N_μ to the condensation of an associated scalar field. Such a condensation is organised in such a way as to emerge from a dynamical coupling of a (real) scalar field ϕ with the internal vector N_μ. We then study the massive excitations around the scalar condensate and relate the corresponding scale of mass to the present observational bound on the cosmological constant.
To arrive at the dynamical coupling of ϕ with N_μ, we start with the current

$$J_\mu = -\frac{1}{2}\phi \overleftrightarrow{\partial_\mu} \phi^{-1}. \tag{2}$$

It can simply be checked that

$$\partial_\mu J^\mu = \phi^{-1}(\Box\phi - \phi^{-1}\partial_\mu\phi\partial^\mu\phi) \tag{3}$$

and

$$J_\mu J^\mu = \phi^{-2}\partial_\mu\phi\partial^\mu\phi \tag{4}$$

are valid. Putting these relations together, we get the identity

$$\Box\phi + \Gamma\{\phi\}\phi = 0, \quad \Gamma\{\phi\} = -J_\mu J^\mu - \partial_\mu J^\mu \tag{5}$$

In what follows we shall call $\Gamma\{\phi\}$ the dynamical mass term.
It should be emphasised that the identity (5) is a formal consequence of the definition (2) and not a dynamical law for ϕ. From it, however, a large class of dynamical theories can be obtained in the form of a divergence theory by making various assumptions about the current J^μ in the dynamical mass term in (5). For example a simple model theory may be characterised by requiring

$$\partial_\mu J^\mu = 0. \tag{6}$$

It leads, as can simply be checked, to a cancellation of the dynamical mass term by the field redefinition $\sigma = \ln\phi$. However, to allow for a dynamical coupling of ϕ with the internal vector N_μ, the dynamical mass term must take a more complicated structure. In this case, we may study a model theory based on a divergence theory of the type

$$\partial_\mu J^\mu = N_\mu N_\nu J^\mu J^\mu \tag{7}$$

Given a divergence theory of the type (7), we get from the identity (5) the corresponding equation for ϕ

$$\Box\phi - (J_\mu J^\nu + N_\mu N_\nu J^\mu J^\nu)\phi = 0 \tag{8}$$

We remark that the dynamical mass-term in (8), is invariant under both transformations[4]

$$\phi \leftrightarrow \frac{1}{\phi}, \quad N_\mu \leftrightarrow -N_\mu \tag{9}$$

[4]Here we treat the scalar field as if it were a dimensionless quantity. In the natural set of units a combination of ϕ and the universal length l_0 may be used to get from ϕ a field having the typical dimension of an inverse distance.

which can be performed independently. This indicates an inherent ambiguity in the theory, because apparently independent configurations composed of ϕ and N_μ become mathematically interchangeable at any physical scale of mass which can be predicted by the dynamical mass-term. In a simple generalisation this ambiguity can significantly be avoided by admitting the universal length l_0 to enter the source of the divergence in (7). We shall study a theory of the type

$$\partial_\mu J^\mu = N_\mu N_\nu J^\mu J^\mu - l_0 N_\mu N_\nu N_\gamma J^\mu J^\nu J^\gamma \tag{10}$$

From the identity (4) we get now the field equation

$$\Box\phi - (J_\mu J^\nu + N_\mu N_\nu J^\mu J^\nu - l_0 N_\mu N_\nu N_\gamma J^\mu J^\nu J^\gamma)\phi = 0 \tag{11}$$

The interesting point to observe is that the invariance property of the dynamical mass-term in (11) connects now both transformations in (9). That is, the dynamical mass term in (11) becomes now invariant under a duality transformation connecting the interchange of ϕ to the reciprocal value ϕ^{-1} to a corresponding reversal of the direction of the internal vector N_μ. The emergence of this duality is considered as reflecting the essential feature of the brocken phase of Lorentz invariance at length scales $\sim l_0$.

It is not unreasonable to expect that 'macroscopic' duality becomes significantly unstable. For example, at distances larger than the universal length l_0 the average value of the scalar field is likely to couple with the matter in the universe. Such a coupling has, among other things, to introduce a duality breaking to single out a preferred configuration composed of the scalar field ϕ and the internal vector N_μ throughout the space. Although the nature of such a duality breaking seems to be significantly linked with the presence of matter and a non-trivial gravitational field [5], it is nevertheless instructive to study its effect on the dynamical theory defined by (10) and (11) in Einstein-Minkowski space.

We shall consider the simplest duality breaking, a preferred back-ground value $\bar{\phi}$ as an average value of ϕ taken over large distances, and correspondingly a preferred back-ground direction of the internal vector N_μ as an average value $\bar{J}_\mu$ of the current J_μ taken over large distances. The essential feature of such a duality breaking is that it requires (on dimensional grounds) that the duality breaking parameters $\bar{\phi}$ and N_μ be interrelated via a relation of the type

$$\bar{J}_\mu = \frac{\lambda}{l_0} N_\mu. \tag{12}$$

where λ can depend only on $\bar{\phi}$. Since we wish to consider the duality breaking as a large distance effect, the characteristic scale of mass defined by the righthand side of (12) must be related to those scales of lengths which are significantly larger than the universal length l_0. This is only the case if λ is taken as significantly small. We assume it to be of order of the ratio of the universal length l_0 and the radius of the universe R

$$\lambda \sim \frac{l_0}{R}. \tag{13}$$

In this way the duality breaking is considered as a cosmological effect. Having assumed a duality breaking of this type, we proceed now to compute the effective form of the

dynamical mass-term in (11) for the divergence theory (10).
First, we may linearise the quadratic term in J_μ in the source of the divergence (10) to find the approximation

$$\partial_\mu J^\mu = N_\mu N_\nu \bar{J}^\mu J^\mu - l_0 N_\mu N_\nu N_\gamma \bar{J}^\mu J^\nu J^\gamma \tag{14}$$

Correspondingly, the quadratic term in the dynamical mass-term of the field equation (11) may be linearised in J_μ to yield

$$\Box\phi - (\bar{J}_\mu J^\nu + N_\mu N_\nu \bar{J}^\mu J^\nu - l_0 N_\mu N_\nu N_\gamma \bar{J}^\mu J^\nu J^\gamma)\phi = 0 \tag{15}$$

Since the background value of J_μ is of the order of λ, the dynamical mass-term can effectively be determined to third order of λ. To this aim, we truncate the non-linear term in J_μ from the source of the divergence (14). Computing then the remaining linear term by means of (12) we get

$$\partial_\mu J^\mu \simeq \frac{\lambda}{l_0} N_\mu J^\mu \tag{16}$$

Now, using (2), this can be written in terms of ϕ as

$$\partial_\mu J^\mu \simeq \frac{\lambda}{l_0} N_\mu \frac{\partial^\mu \phi}{\phi}. \tag{17}$$

The right hand side of this equation can be linearised in ϕ by using in the dominator the background value $\bar{\phi}$ for ϕ. An approximate solution of (17) which is compatible with (12) can then be given

$$J_\mu \simeq \frac{\lambda}{l_0} \frac{\phi}{\bar{\phi}} N_\mu. \tag{18}$$

We now use this solution for J_μ in (15) to arrive at the field equation

$$\Box\phi - (2\frac{\lambda^2}{l_0^2}\frac{\phi}{\bar{\phi}} - \lambda \frac{\lambda^2}{l_0^2}\frac{\phi^2}{\bar{\phi}^2})\phi = 0 \tag{19}$$

in which a linear and a quadratic term in ϕ appears in the dynamical mass-term. We can get a more effective form of this equation if we use in the linear term the back-ground value $\bar{\phi}$ for ϕ, leading to

$$\Box\phi - (2\frac{\lambda^2}{l_0^2} - \lambda \frac{\lambda^2}{l_0^2}\frac{\phi^2}{\bar{\phi}^2})\phi = 0. \tag{20}$$

This shows the effective contribution of the divergence theory (10) to the dynamical mass-term in (11). Equation (20) can be derived from the effective Lagrangian density

$$\mathcal{L} = \frac{1}{2l_0^2}[\partial_\mu \phi \partial^\mu \phi - V(\phi)], \quad V(\phi) = -2\frac{\lambda^2}{l_0^2}\phi^2 + \frac{1}{2}\lambda \frac{\lambda^2}{l_0^2}\frac{\phi^4}{\bar{\phi}^2} \tag{21}$$

from which we can now define the ground state value ϕ_0 of ϕ by minimising the potential $V(\phi)$. This gives the condition $\phi_0^2 = \frac{2}{\lambda}\bar{\phi}^2$. Let us choose $\phi_0 = (\frac{2}{\lambda})^{1/2}\bar{\phi}$ as the ground state

value on which to study the nature of physical excitations. The potential $V(\phi)$ can be expanded around ϕ_0 to yield (neglecting constant terms)

$$V(\phi) = 8(\frac{\lambda}{l_0})^2(\phi - \phi_0)^2 + O((\phi - \phi_0)^3), \tag{22}$$

from which we infer that physical excitations of ϕ around the ground-state value ϕ_0 provides a characteristic scale of mass of the order $\sim \frac{\lambda}{l_0}$.
We may therefore argue that in the broken phase of the Lorentz invariance an effective cosmological constant Λ must appears which receives contributions proportional to $(\frac{\lambda}{l_0})^2$, namely

$$\Lambda \sim (\frac{\lambda}{l_0})^2 \tag{23}$$

which in conjunction with (13) yields

$$\Lambda \sim \frac{1}{R^2}. \tag{24}$$

The agreement of this relation with the present observational bound for the cosmological constant is a remarkable consequence of the well-known empirical fact that the present universe has just the characteristic size $R \sim 10^{29} cm$.

VI-Concluding remarks

In this note we have demonstrated that the broken phase of Lorentz invariance can provide a consistent contribution to the cosmological constant via the mass scale of an associated scalar condensate. The basic input was to consider the duality breaking as a cosmological effect.
We emphasize that there may be a potential dependence of the energy density of the vacuum on the entire mass spectrum of physical fields in quantum field theory, for example the Higgs mass. We have not commented on the dynamical reasons for why the corresponding contributions should cancel out. In this respect we must still look for more or less natural rules to establish the applicability of the model presented for the prediction of the value of the cosmological constant. We hope to address the issue elsewhere another publication.

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